

A METHOD FOR COMPARISON TESTING OF WINDOW ACCESSORIES

T.C. Lexen P.F. Muldary

ABSTRACT

A test facility has been developed for the comparison of the thermal performance of window accessories. The facility is presented as a developmental tool, allowing direct comparison testing of prototypes and new products, under controlled interior and real weather conditions. Testing for differences in U-value, net solar gain and 24-hour net gain between window systems is described. Error analysis and testing limitations are discussed, along with testing options and future plans.

INTRODUCTION

Window accessories such as shades, blinds, films, shutters, and movable insulation generally function to help conserve energy in a building by controlling heat losses and/or gains through windows. A particular accessory will thus improve the thermal performance of a window, primarily by reducing night loss or solar gain. Development of our facility began in mid-1982 with the need for a test setup capable of thermal performance testing to support the development of new products--window accessories--to improve the overall performance of a window.

Current industry standards for assessing thermal performance of windows and window accessories concern three basic properties: U-value, shading coefficient and air infiltration. The properties are determined in separate laboratory tests. The expense involved in setting up to perform these tests in-house or having an independent lab run the tests would have been prohibitive to a small manufacturer. Limited testing and time delays would have restricted our design options and hampered our ability to develop progressive, well-tested products. As a result we sought a better means of testing.

A test method was developed that is simple in concept, inexpensive to install and operate, and flexible in configuration and operation. It allows the measurement of the above-mentioned improvement in thermal performance of a window and the comparison of accessories under real weather conditions. This facility has been in operation since April 1983 and is still in the developmental stage. Improvements are being made in hardware, test methods and accuracy. Future improvements are being investigated as we gain in experience and knowledge. This paper presents the facility as it currently operates.

CONCEPT

It is necessary for direct comparison testing such as ours to insure that competing specimens are subjected to identical conditions. One tries to isolate the differences of the specimens, so that the differences in test system performance can be attributed to differences in the specimens only.

Timothy C. Lexen is Product/Text Engineer, AMSCO Building Products, Rice Lake, WI;
Patrick F. Muldary is Manager of Engineering, AMSCO Building Products, Rice Lake, WI.

The test method presented here involves mounting window specimens in identical test cells¹ that are subjected to identical interior conditions and prevailing weather conditions. By comparing the heating powers required to maintain the same temperature in each cell, we are able to determine the difference in heat flow between two specimens. Then we can calculate the difference in U-value, net solar gain or 24-hour net gain due to an accessory or between window systems. This test method is based on the assumption that the cells, minus the window areas, behave identically. While the absolute performance values calculated from the tests are affected by prevailing conditions, the difference in performance between simultaneously tested specimens should allow a realistic comparison.

TEST APPARATUS

The facility consists of three test cells, a weather station and an instrument center. The test cells are small buildings, with interior dimensions 8 ft (2.4 m) high by 8 ft (2.4 m) wide by 4 ft (1.2 m) deep, mounted on the roof of our manufacturing plant. Each building is of conventional 2 x 4 frame construction, covered with plywood siding. The walls, floor and roof are insulated with 6.5 in (0.16 m) of polyisocyanurate foam, yielding R-32. Interior surfaces are fiberglass panels sealed with resin. Average cell insulation, for all but the window area, including an insulated and weather-stripped door, is R-30.

The stud wall on the front of each cell can be removed and framed to accommodate any window, or cluster of windows, up to 8 ft x 8 ft (2.4 m x 2.4 m). Window size at present is 13.4 ft² (1.2 m²): 42% of floor area, and 5.5% of total interior area.

Inside each cell is an electric resistance heater, an air circulation fan and two platinum RTD probes. One RTD provides input to a temperature controller, while the other feeds a dot recorder. Double concentric tube radiation shields are used for measurements in direct sunlight.

Also atop the plant is a weather station consisting of wind vane, anemometer, precision thermistor and precision thermopile pyranometer. The latter measures total solar radiation (direct & diffuse) in the south facing vertical plane.

The instrument center houses the control and data acquisition equipment. Each cell has a proportional, integral, derivative (PID) controller maintaining cell temperature to within 0.3 F (0.54°C) of set point. Each controller varies the power to its heater via a solid state contractor (note that no auxiliary cooling is available). Power used by each cell is monitored using an AC watt transducer with Halls effect multiplier. Cell power and weather data are collected and stored in a data logger that has disk drive, plotter and printer. Data are later retrieved, plotted, and analyzed. A six-channel dot recorder plots cell interior temperatures for monitoring purposes.

TEST METHODS

We perform three types of testing in order to get three performance parameters: (1) a night test provides U-value and differences in U-values, (2) a daytime test provides net solar gain and differences in net solar gain, and (3) a 24-hour test yields 24-hour net gain and differences therein. The three tests are run in a near-identical manner; a 24-hour test can be analyzed to extract the three parameters.

Since the test methods and principles are the same for the three tests, and to simplify an otherwise complex discussion, the night U-value test will be described and discussed at length with the daytime and 24-hour test discussed only briefly, emphasizing their unique contributions and problems. This will also put a greater emphasis on the area of testing in which we have the most experience and the greater confidence in our methods and accuracy.

NIGHT U-VALUE AND U TEST

The setup typically is as follows: the first cell is a reference cell, with a high resistance (R-40) thermal plug instead of a window; the second cell has a standard double-pane window

¹Also called passive solar test cells or test rooms (Moore and McFarland 1982).

(R-2); and the third cell has an identical standard window with accessory. The cells face the same direction, usually north, to avoid daytime overheating. Interior temperature is 72 F (22.2°C). (Note: in summer, inside temperature is elevated to maintain a reasonable temperature difference across the window. Tests are being conducted to determine the effect of elevated temperature on results).

An overnight test runs roughly from sunset to sunrise. The sampling interval of the data logged is usually set at 14.4 seconds for all seven channels.

After the test, the seven channels of data are retrieved, plotted with colored pens, and visually inspected for trends or problem areas. Figure 1 shows a plot from a typical night test, from 6 p.m. to 6 a.m., in springtime. The numerical data can be extracted for any point or time interval. We look at the data for the period 3 a.m. to 5 a.m., when wind, outside temperature and cell behavior are most stable. Also, by this time, the cell walls have lost any stored heat gained by absorbing sunlight during the day. Cell power means and ambient temperature mean for the two-hour period are used in the calculations shown below.

THEORY

Imagine a boundary envelope just under the inner surface of the floor, door, walls and ceiling, excluding the window area. Together the window and envelope form a system boundary, and we can perform an energy balance on the cell. Since the cell remains at constant temperature, the system is steady state and the energy flowing in equals the energy flowing out. For each of the three cells, the simplified equations below show that the measured power in is equal to the heat loss through each envelope and the plug or window

$$Q_1 = Q \text{ envelope}_1 + Q \text{ plug} \quad (1)$$

$$Q_2 = Q \text{ envelope}_2 + Q \text{ window}_2 \quad (2)$$

$$Q_3 = Q \text{ envelope}_3 + Q \text{ window}_3 \quad (3)$$

Where Q is heat flow rate in Btu/h(W).

Assuming for the moment that envelope losses are equal for Cells 1 and 2, subtracting yields

$$Q_2 - Q_1 = Q \text{ window}_2 - Q \text{ plug} \quad (4)$$

$$Q \text{ window}_2 = (Q_2 - Q_1) + Q \text{ plug} \quad (5)$$

(Q plug is about five percent of Q window and thus is not negligible; it can be calculated from the plug's known R-value.)

Average U-values are then calculated for the two-hour period using

$$U = Q \text{ window} / (A_w \cdot \Delta T_{10}) \quad (6)$$

where

U = 2-hour average thermal transmittance,

in Btu / (h · ft² · F), (W / m² · K)

A_w = area of window, ft² (m²).

ΔT₁₀ = Temperature difference across window,

= (T_{inside} - T_{outside}), F (°C).

In practice, we calculate Q window and U-value as if the plug loss were negligible, then add the U-value (= 1/R) of the plug; this is the equivalent of dividing Equation 5 by A_wΔT₁₀ to get

$$U \text{ window} = U_{21} + U \text{ plug} \quad (7)$$

Error incurred with this approach is discussed in Appendix B.

In a similar manner, the difference in U-values, ΔU can be calculated from

$$\Delta Q_{2-3} = Q_2 - Q_3 = Q_{\text{window2}} - Q_{\text{window3}} \quad (8)$$

$$\Delta U = \Delta Q_{2-3} / (A_w \cdot \Delta T_{i,o}) \quad (9)$$

Again, ΔU is a two-hour average in Btu / (h · ft² · F), (W / m² · K). Note that the plug heat loss does not enter into these equations.

These U and ΔU values are compared with values from the same period on other nights, possibly from a round-robin test arrangement. If the test conditions warrant, ΔU and U-values can be checked for other two-hour periods in the night or for any point or interval. Also, the data logger can be programmed to calculate U and ΔU on a point-by-point basis (rather than using means) and plot them against time.

Note that for both U and ΔU we are measuring a difference in heat loss through two window areas; in calculating U, one of those window areas contains a plug with heat loss that is known or negligible.

USE OF U AND ΔU

In comparing our results with those from standard laboratories, we face problems common to other field facilities (McCabe 1985). First, our interior conditions are different from laboratory conditions in terms of convection currents, surrounding surfaces and their temperatures, air pressures, etc. Second, our outside conditions are both uncontrolled and highly variable, so we get quite a bit of scatter in our U-values and somewhat less scatter in the ΔU measurements.

Our emphasis is on measuring the difference in U-value, ΔU . It represents the difference in heat loss between the window alone and the window-with-accessory, under particular weather conditions. We can say that the accessory reduces the standard window's U-value by ΔU . The effectiveness of the accessory can be expressed as a percent reduction in heat loss (from a standard window), as is done in laboratory tests such as WES 1584 and the guarded window test apparatus (Andersen 1982). Care must be taken in determining the standard window's U-value, and one must realize that the ratio $\Delta U/U$ varies with wind conditions.

Recall that ΔU for an accessory has meaning only in the context of the window; ΔU is not a property of the accessory alone, but describes the effect of the accessory on a particular window. Because of the way heat flows through parallel resistances is calculated, the measured ΔU for a given accessory will be different when tested on single- and double-pane windows. Appendix A lists the several standard test methods for measurement of U-value or difference in U-value.

ACCURACY

Since we are presenting a test method rather than a specific test, we will deal here with the question of accuracy in our test results. Much recent effort has gone into internal testing and error analysis. In our error analysis, sources of error (assumed random) are identified and combined (Klems 1984). These include estimates of error due to air infiltration, interior temperature fluctuations, and measurement of the thermal plug's U-value; these are comparatively small.

For typical conditions, we estimate that we can measure the U-value of a double-pane window ($U = 0.5$ (2.8)) with an accuracy (i.e., with error) in the range of ten to fifteen percent. We estimate that we can measure a ΔU of 0.2 (1.1), on a double-pane window, with an accuracy in the range of twenty to twenty-five percent. The error estimates are affected by test conditions and are nearly inversely proportional to U or ΔU . These estimates should be considered approximate lower bounds on errors.

The above total errors are dominated by errors in measurement of the cell power two-hour averages. Typically, the standard deviation (RMS error) on the power average is three percent of total power. This small error becomes a larger percentage of the difference in cell powers; the absolute error stays the same even though the difference in cell power is small. This is a problem inherent in taking the small difference between two large measurements.

The above total errors also include errors due to differences in envelope losses; the envelope losses in Equations 1 through 3 are not identical. As part of our test procedure, we check the "balance" of the cells by installing thermal plugs in all three cells and running a test overnight or for several nights. We then compare cell power averages to see how close the cells are. Typically, the maximum difference in two-hour means between any two cells is two percent. This includes heat loss due to air infiltration.² This two percent error is assumed for night testing. We assume that additional differences in envelope loss, due to differences in effects of plug and windows, are negligible. These latter could include differences in convection currents, radiation exchanges, or wall temperatures; in sunlight testing, these could be factors in our results and errors.

Perhaps the best use of our error analysis is to determine the limits within which we can do acceptably accurate testing. For example, if we set an accuracy requirement of fifteen percent on measurements of ΔU on a double-pane window, the effective ΔU of an accessory should be greater than 0.25 - 0.30. Note that the above estimated errors and testing limits for ΔU are dependent on the R- or U-value of the standard window on which the accessory is tested. An accuracy requirement of fifteen percent on measured U requires that we test window systems with a U-value of at least 0.3 (2.8), (R<3).

Recall that our primary interest is the testing and comparison of prototypical accessories as part of the design process, rather than testing for absolute performance of components. We can tolerate errors greater than fifteen percent.

Replacing the plug in Cell 1 with a window specimen gives us an important option. One standard window can be run against windows with two different accessories, and ΔU calculated for each of the accessories; or identical windows can be fitted with three different accessories and ΔU calculated for the differences between the window systems. This type of testing is ideally suited to new product development, where two or three prototypes are run side by side, or prototypes are compared to existing products. For night testing, loss from a plugged cell can be estimated using its overall loss coefficient (net load coefficient of about 8.5 Btu/hr. - F). The same cannot be done for daytime testing due to heat absorbed and stored in the walls and roof.

NET SOLAR GAIN TEST

Set-up is the same as for night testing--plug, bare window, and accessoried window in Cells 1, 2, and 3--except, that the cells face south. This test requires an electric coupling between the three temperature controllers, known as the "remote set point mode." The set point for Cells 1 and 3 is the actual interior temperature of Cell 2. This insures that the three cells maintain the same temperature even if the temperature of Cell 2 (with bare window) becomes elevated. Cell 2 is assumed to receive the larger solar gain and the greater temperature elevation.

The behavior of the three cells on a sunny day, with the accompanying explanations and analysis, is quite complex and beyond the scope of this paper. In summary, we perform an energy balance on each cell and derive equations paralleling Equations 1 through 5 with the same assumptions, but ignoring the plug loss, where

$$Q_1 - Q_3 = \text{net solar gain of window and accessory}$$

$$Q_1 - Q_2 = \text{net solar gain of window alone}$$

$$Q_3 - Q_2 = \text{difference in net solar gain, due to accessory}$$

This net solar gain consists of the solar energy coming in and the heat loss going back out the window; it can be positive or negative. A shading coefficient for the accessory can be determined by using a previously determined nighttime U-value and the pyranometer reading, but this seems to us to be working backward. We prefer to calculate the ratio of the net solar gain of the window-with-accessory to that of the standard window alone, what we call the shading factor. This ratio expresses the accessory's reduction of the net solar gain of the window.

² Cell air leakage for each cell envelope alone is about 0.8 a.c.h.; cells balance to within about 0.05 a.c.h. Window leakage is additional.

One probable source of error is dependent on the type of shading accessory we test, whether (1) transparent or tinted, or (2) translucent or opaque. Transparent or tinted accessories, as well as bare windows, let sunlight through to hit the cell's inside walls and floor. Some fraction of the solar energy incident on these surfaces is absorbed and flows out of the building (Klems 1984, pp. 11, 22). If this fraction is significant, our assumption of identical envelope losses for the three cells (plugged, with window, and with accessory) is invalid. While the plugged reference cell and accessoried cell kept the same interior air temperature as the bare window cell, their envelope losses would be less. LBL's MoWITT facility and other calorimeters use effective solutions to this problem, but we are considering some simple, low-cost solutions. We are installing a black felt curtain across the middle of each cell in an attempt to "thermalize" the incoming radiation, absorb it, and transfer the heat to room air. Tests are planned to check the effectiveness of this approach.

Much more research is needed to clarify our error analysis for daytime testing. We expect that, as for night testing, we will be able to accurately measure differences in performance above some minimum difference.

24-HOUR NET GAIN TEST

For a 24-hour period, each of the cell power curves is integrated to get the area under the curve, the total energy added to maintain cell temperature. Subtracting the total energy added to the plugged cell from the total energy added to the windowed cell yields the net heat gain of the window system. Also, subtracting the total cell energy of two windowed cells yields the difference in 24-hour net gain of the window systems due to the accessory. From this, it can be seen whether, and how much, an accessory improves the 24-hour performance of a window. This latter information may be the most valuable that can be gained using this test facility.

OTHER TESTS

Although this paper has emphasized testing window accessories, the above methods can be applied to comparing windows of various designs and materials.

In addition to the options described above, we have equipment and the flexibility to investigate issues of interest. We can monitor 1) relative humidity, for condensation testing, 2) transmitted ultraviolet light, and 3) heat buildup between window and accessory in any cell. We can vary the internal characteristics of the cells and study the effects. We can introduce controlled air infiltration in our tests, as well as run static tests, by means of equipment on each cell. (Unless measures are taken to seal each window, U and ΔU include air infiltration caused by normal wind conditions.) The cells can be rotated to face any horizontal direction.

FUTURE PLANS

In addition to testing to support new product development, our main goal and activity, we plan the following:

1. Improve our facility and test methods. This will include adding cell insulation, reducing cell air leakage, and installing the solar-absorbing black felt curtains mentioned earlier.
2. Further define the error analysis, particularly for our daytime and 24-hour testing.
3. Possible future tests include comparisons of existing window systems and accessories, long-term performance comparisons, testing new window systems such as low-E glass, and studying the effect of wind and other variables on window performance.

CONCLUSION

The facility described in this paper has been successful at meeting our initial objective of providing a method to allow accurate and realistic comparison testing for product development. Measurement of differences in the thermal/solar performance of window systems and window accessories can be made with acceptable accuracy. The system is flexible in configuration and operation and could be used in researching pertinent window energy concerns. The facility has the potential to accurately compare the 24-hour net performance (gains or losses) of window

systems and prototypes. More research is needed to further define error analysis for both the net solar gain and the 24-hour net gain tests.

REFERENCES

- Anderson, J.J. 1982. "Guarded window test apparatus for determining the insulation value of interior window coverings." Textile Research Journal, Vol. 52, No. 10, pp. 643-651.
- Klems, J.H. 1984. "Measurement of fenestration net energy performance: considerations leading to development of the Mobile Window Thermal Test (MoWITT) facility." Berkeley: Lawrence Berkeley Laboratory. LBL-18144.
- Lawrence Berkeley Laboratory. 1981. "A comparison of products for reducing heat loss through windows." U.S. Department of Energy. Report #W-73.
- McCabe, M.E. 1985. "Comparison between laboratory and field measured U-values for a triple-glazed window." Workshop on Laboratory Measurement of the U-values of Windows. Gaithersburg: National Bureau of Standards.
- Moore, E.F.; McFarland, R.D. 1982. "Passive-solar test modules." Los Alamos: Los Alamos National Laboratory. LA-9421-MS.

BIBLIOGRAPHY

- ASHRAE. 1981. ASHRA Handbook--1981 Fundamentals. Chapter 27. Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.
- ASTM. 1981. ASTM Method C 236-80, "Steady-state thermal performance of building assemblies by means of a guarded hot box." Philadelphia: American Society for Testing and Materials.
- Hyde, J.C. 1981. "Passive test cell experiments during the winter of 1979-80." Los Alamos: Los Alamos National Laboratory. LA-9048-MS.
- Selkowitz, S.E. 1979. "Thermal performance of insulating window systems." Berkeley: Lawrence Berkeley Laboratory. LBL-8835.
- WES. 1984. WES 1584, "Test procedure for thermal transmittance of window treatments and movable insulation." St. Paul: Window Energy Systems Division, Industrial Fabrics Association International.

APPENDIX A

Laboratory Tests for U and ΔU :

1. U-value alone:

- ASTM C236-80 Guarded Hot Box
- ASTM C976 Calibrated Hot Box
- AAMA 1503.1 Window and Door Test Method
- BS 874 British Standard on Thermal Measurements
- ISO/TC 163 Calibrated and Guarded Hot Box

2. Specifically for ΔU due to accessories:

- WES 1584 Guarded Hot Box
- Window Heat Transfer Apparatus (With Heat Flux Transducers); Reference: Vol. 51, No. 7, 7/81, Textile Research Journal.
- Guarded Window (Guarded Hot Plate); Reference: Vol. 52, No. 10, 10/82, Textile Research Journal.

APPENDIX B

Nomenclature:

E	cell envelope area (i.e., minus the window area), ft ²
F	window area, ft ²
H	envelope heat loss, Btu/hr
I	heat loss due to air infiltration, Btu/hr
L _b	auxiliary heat to cell during balancing, Btu/hr
L _c	auxiliary heat to cell during test, Btu/hr
R	envelope R-value, (hr-ft ² -F)/Btu
T _i	inside air temperature, F
T _o	outside air temperature, F
ΔT _{io}	temperature difference (air-to-air), F
U	thermal transmittance (air-to-air), U-value, Btu/hr-ft ² -F
ΔU	difference in U between two window systems, Btu/hr-ft ² -F
W	net heat flow through the window area, outward, Btu/hr
ΔW	difference in net heat flow, Btu/hr
δ	measurement error (in specific quantity)

ERROR ANALYSIS FOR NIGHT U MEASUREMENTS

The following error analysis is done for measurements of ΔU (U-value measurement being a special case of ΔU measurement) under our typical test condition; that is, 3-5 a.m., when ambient conditions are fairly calm and stable. This is the closest we get to steady state conditions. We will consider using two-hour averages of data to get an average ΔU.

Consider a single cell with installed double-pane window. Under steady state conditions, we can perform a heat flow balance on the cell which says that the window heat flow is

$$W = CVdT/dt - H - I + L_c \quad (1)$$

Where L_c is auxiliary heat added to the space; W, H, and I are heat flows out of the space (Klems 1984).

The term CVdT/dt denoted the change in heat content of the air due to variation of the temperature (of air and cell inside surfaces) with time. Since cell temperature stays within ± 1/4 F during testing, this term is negligible and will be ignored in the remaining discussion.

The fractional error in W is given by

$$\frac{\delta W}{W} = \left\{ \left[\frac{L_c}{W} \left(\frac{\delta L_c}{L_c} \right) \right]^2 + \left[\frac{H}{W} \left(\frac{\delta H}{H} \right) \right]^2 + \left[\frac{I}{W} \left(\frac{\delta I}{I} \right) \right]^2 \right\}^{0.5} \quad (2)$$

Now consider two cells, each with a different window system, where Cell #2 has a window with greater heat loss:

$$W_2 = L_{c2} - H_2 - I_2 \quad (3)$$

$$- W_1 = L_{c1} - H_1 - I_1 \quad (4)$$

$$\frac{W_2 - W_1 = L_{c2} - L_{c1} - (H_2 - H_1) - (I_2 - I_1)}{\quad} \quad (5)$$

$$\Delta W_{21} = (L_{c2} - L_{c1}) - [H_2 - H_1 + I_2 - I_1] \quad (6)$$

where L_{c2} and L_{c1} are measured directly. Note that we are focusing on differences in auxiliary power, envelope loss and air infiltration loss.

We can measure air leakage for each cell (in a.c.h.) and calculate I_1 and I_2 , and also determine the difference in envelope losses by balancing the cells using plugs. But since the plugs do not interfere with window air leakage, we can balance the cells under representative wind conditions to get the quantity $[H_2 - H_1 + I_2 - I_1]$ in Equation 6. If we let P be plug heat loss and L_b the auxiliary heat during balancing for the two cells:

$$L_{b2} - P_2 + H_2 + I_2 \quad (7)$$

$$- L_{b1} = P_1 + H_1 + I_1 \quad (8)$$

$$\frac{L_{b2} - L_{b1} = (P_2 - P_1) + (H_2 - H_1 + I_2 - I_1)}{\quad}$$

Since plug loss is only four percent of envelope loss, we will consider the difference in plug loss (some fraction of one percent) to be negligible. Then

$$L_{b2} - L_{b1} = (H_2 - H_1 + I_2 - I_1) \quad (9)$$

where the right-hand side is the quantity sought in Equation 6; the difference in envelope losses, including infiltration differences. Rewriting Equation 6:

$$\Delta W_{21} = L_{c2} - L_{c1} - (L_{b2} - L_{b1}) \quad (10)$$

The difference $L_{b2} - L_{b1}$ is approximately zero; we will be concerned with $\delta(L_{b2} - L_{b1})$. The fractional error in ΔW_{21} is

$$\frac{\delta \Delta W_{21}}{\Delta W_{21}} = \left\{ \left[\frac{L_{c2}}{\Delta W_{21}} \left(\frac{\delta L_{c2}}{L_{c2}} \right) \right]^2 + \left[\frac{L_{c1}}{\Delta W_{21}} \left(\frac{\delta L_{c1}}{L_{c1}} \right) \right]^2 + \left[\frac{\delta(L_{b2} - L_{b1})}{\Delta W_{21}} \right]^2 \right\}^{0.5} \quad (11)$$

Columns 1 and 2 of Table 1 show these three error sources. Column 3 develops the three parts in terms of the window and facility parameters. In the first two rows of column 3, $\delta L/L$ is left intact to be kept as a percentage error. In all rows ΔW_{21} is the difference in heat loss between windows:

$$\Delta W_{21} = \Delta U (F) \Delta T_{i0} \quad (12)$$

L_{c2} and L_{c1} are each the sum of envelope loss and window loss:

$$L_{c1} = E T/R + U_i F T \quad (13)$$

where E = envelope area
 F = window area
 R = envelope R-value

When we balance the two cells with plugs, the difference in mean powers varies; it is not a constant difference between any two cells (say, from night to night, or between any time periods). It averages out to be approximately zero; however, the largest difference in mean powers between any two cells is about two percent of total power. We say that the cells balance, but with an error of two percent; i.e., that difference in two-hour means could be as large as two percent of total power. So, $\delta(L_{b2} - L_{b1})$ is two percent of envelope loss, or

$$\delta(L_{b2} - L_{b1}) = 2\% \text{ of } E \Delta T / R \quad (14)$$

Additional differences in envelope loss due to difference in convection currents, wall temperature or radiation exchanges are considered negligible.

Power to each electric heater varies considerably to keep inside temperature within tight limits. This can be seen in the power curves of Figure 1. On the average, this is about three percent of total power; i.e., $\delta L_c / L_c = 3$ percent for each cell.

In Column 4, we look at the error on measurement of a ΔU between a double-pane window ($U_2 = 0.5$) and a window with accessory ($U_1 = 0.3$), for a total error of 22 percent.

The following values are used:

$$E = 233.3 \text{ ft}^2$$

$$F = 13.4 \text{ ft}^2$$

$$R = R - 30$$

Note that for each contribution, and thus for the total, the error is inversely proportional to ΔU .

U-VALUE

In order to get U-value and its associated error by the above calculations, we must backtrack to Equation 4. Putting a plug in Cell #1, we have $W_1 = \text{plug loss} = W_p$.

We are looking for W_2 , the window net heat flow, in order to calculate U-value. We find by revising Equation 6,

$$W_2 = L_{c2} - L_{cl} - (L_{b2} - L_{b1}) + W_p \quad (15)$$

so that within Equation 11 we must add the term

$$\dots + \left[\frac{W_p}{\Delta W_{21}} \left(\frac{\delta W_p}{W_p} \right) \right]^2 \Bigg]^{0.5}$$

Rewritten in terms of cell characteristics, this becomes

$$\dots + \left[\frac{U_p}{U} \left(\frac{\delta W_p}{W_p} \right) \right]^2 \Bigg]^{0.5} \quad (16)$$

At present, we are using plugs of R-40 (based on calculation from material properties). Thus the plug's heat loss is roughly 5 to 7.5 percent of the loss from a double-pane window ($U = 0.3$ to 0.5). Assuming an error of ten percent in estimating the plugs R-value, we have an error contribution of less than one percent.

Column 5 presents the error contributions for measurement of U-value of a window ($U_2 = 0.5$) when the plug loss is negligible ($U_1 \cong 0$).

OTHER SOURCES OF ERROR

Note that this error analysis deals only with ΔW , not $\Delta U = \Delta W / A \Delta T_{i0}$. This is because the error in ΔT_{i0} is very small and negligible. Because of the way resultant error are calculated for the case of division, unless δT_{i0} is large (at least 20 percent of ΔT_{i0}) its effect on the resultant error is negligible. Errors on T_i and T_o figured together for ΔT_{i0} are in the range of two to three percent.

Because of the way the error source contributions are summed--by squaring them first--small errors become insignificant. Errors less than 20 percent of the largest error do not contribute significantly.

However, if we do testing so that total error is less than ten percent, we should consider taking into account errors otherwise considered negligible. Thus, we would expect that the error analysis "bottoms out" anywhere under ten percent due to the small error contributions which then become significant.

TABLE 1 - Error Source Contributions

(1)	(2)	(3)	(4)	(5)
Source	Terms	$\frac{\delta U}{\Delta U}$ calculations	$U_2 = 0.5$ $U_1 = 0.3$ $\Delta U = 0.2$	$U_2 = 0.5$ $U_1 \cong 0$ $\Delta U = 0.5$
Auxiliary Power	$\frac{L_{c2}}{\Delta W_{21}} \left(\frac{\delta L_{c2}}{L_{c2}} \right)$	$\frac{1}{\Delta U} \left(\frac{E}{RF} + U_2 \right) \frac{\delta L_{c2}}{L_{c2}}$	$\frac{1}{0.2} (.58 + 0.5)(3\%)$ $= 16.2\%$	6.5%
Auxiliary Power	$\frac{L_{c1}}{\Delta W_{21}} \left(\frac{\delta L_{c1}}{L_{c1}} \right)$	$\frac{1}{\Delta U} \left(\frac{E}{RF} + U_1 \right) \frac{\delta L_{c1}}{L_{c1}}$	$\frac{1}{0.2} (.58 + 0.3)(3\%)$ $= 13.2\%$	3.5%
Difference in Envelope Loss	$\frac{\delta(L_{B2} - L_{B1})}{\Delta W_{21}}$	$\frac{1}{\Delta U} \left(\frac{E}{RF} \right) (\delta\%)$	$\frac{1}{0.2} (.58)(2\%)$ $= 5.8\%$	2.3%
Plug Loss	$\frac{W_P}{\Delta W_{21}} \left(\frac{\delta W_P}{W_P} \right)$	$\frac{U_P}{U} \left(\frac{\delta W_P}{W_P} \right)$	X	0
TOTAL			22%	8%

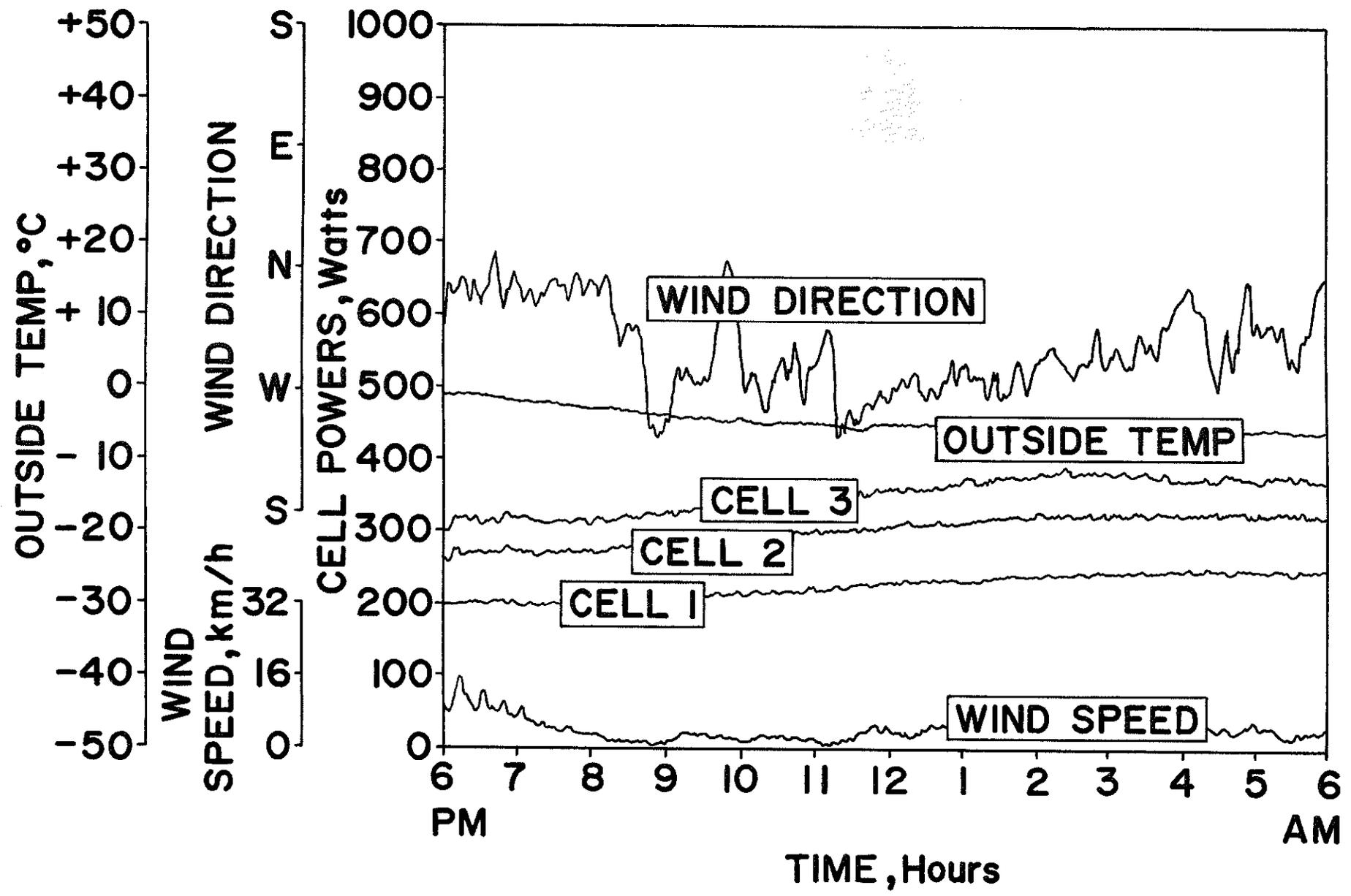


Figure 1. Sample overnight test for U , ΔU